

Seminar

Speaker: [Dimitrios M. Thilikos](#)
(Professor, Department
of Mathematics, UoA)
Optimal Erdős-Pósa

Title: properties for θ_r minor
models

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[Department of](#)
[Mathematics, University](#)
[of Athens, room Γ33](#)

Abstract

Typically, Erdős-Pósa properties reveal relations between covering and packing invariants in combinatorial structures. The origin of the study of such properties comes from the celebrated Erdős-Pósa Theorem (1965), stating that there is a function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that for every $k \in \mathbb{N}$ and for every graph G , either G contains k vertex-disjoint cycles or there is a set X of $f(k)$ vertices in G meeting all cycles of G . In particular, Erdős and Pósa proved this result for $f(k) = O(k \cdot \log k)$ and showed that this bound is optimal. Given a graph J , we denote by $M(J)$ the set of all graphs that can be contracted to J (also called models of J). Robertson and Seymour proved that the class $M(J)$ satisfies the Erdős-Pósa property if and only if J is planar. Notice that this can be seen as the (qualitatively tight) extension of the Erdős-Pósa Theorem (take $J = \theta_2$ where, in general, θ_r is the graph consisting of two vertices and r parallel edges between them). The emerging question is whether (and when) the function involved in the above proposition can match the (optimal) $O(k \log k)$ bound of Erdős-Pósa and whether this bound can be improved under several assumptions on the considered graphs. Given two graphs H and G , we denote by $\text{pack}_H(G)$ as the maximum number of vertex-disjoint models of H in G . We also denote by $\text{cover}_H(G)$ the minimum number of vertices that intersect all models of H in G . We prove the following result. Theorem 1. There exist a function $f : \mathbb{N} \rightarrow \mathbb{N}$ such that for every two positive integers r, q and every graph G excluding K_q as a minor, it holds that $\text{cover}_{\theta_r}(G) \leq f(r) \cdot \text{pack}_{\theta_r}(G) \cdot \log q$. Our proof can be adapted for the edge-variant of the same theorem (where we consider edge coverings and edge-disjoint models). Our results also imply that, for every r , the problems of computing the values of pack_{θ_r} , cover_{θ_r} , as well as their “edge” counterparts admit $\log(\text{OPT})$ -approximation (deterministic and polynomial) algorithms. This improves existing results on the approximability of the above graph invariants. (Joint work with: Dimitris Chatzidimitriou, Jean-Florent Raymond, and Ignasi Sau).