Seminar

Speaker:	Dimitrios M. Thilikos (Professor, Department of Mathematics, UoA)
Title:	Optimal Erd■s-Pósa proterties for θ_r minor models
Date:	Friday, 28 Feb 2014
Time:	18:30-19:30
Location:	Univeristy of Athens, Department of Mathematics, University of Athens, room Γ33

Abstract

Typically, Erd■s–Pósa properties reveal relations between covering and packing invariants in combinatorial structures. The origin of the study of such properties comes from the celebrated Erd■s–Pósa Theorem (1695), stating that there is a function f : N → N such that for every k ∈ N and for every graph G, either G contains k vertex-disjoint cycles or there is a set X of f(k) vertices in G meeting all cycles of G. In particular, Erd■s and Pósaa proved this result for f(k) = O(k · log k) and showed that this bound is optimal. Given a graph J, we denote by M(J) the set of all graphs that can be contracted to J (also called models of J). Robertson and Seymour proved that the class M(J) satisfies the Erd■s–Pósa property if and only if J is planar. Notice that this can be seen as the (qualitatively tight) extension of the Erd■s–Pósa Theorem (take J = θ2 where, in general, θr is the graph consisting of two vertices and r parallel edges between them). The emerging question is whether (and when) the function involved in the above proposition can match the (optimal) O(k log k) bound of Erd■s–Pósa and whether this bound can be improved under several assumptions on the considered graphs. Given two graphs H and G, we denote by packH (G) as the maximum number of vertex-disjoint models of H in G. We also denote by cover_H (G) the minimum number of vertices that intersect all models of H in G. We prove the following result. Theorem 1. There exist a function f : N → N such that for every two positive integers r, q and every graph G excluding K_q as a minor, it holds that cover_θ_r (G) ≤ f (r) · pack_θ_r (G) · log q. Our proof can be adapted for the edge-variant of the same theorem (where we consider edge coverings and edge-disjoint models). Our results also imply that, for every r, the problems of computing the values of pack_θ_r , coverv_θ_r , as well as their “edge” counterparts admit log(OPT)-approximation (deterministic and polynomial) algorithms. This improves existing results on the approximability of the above graph invariants. (Joint work with: Dimitris Chatzidimitriou, Jean-Florent Raymond, and Ignasi Sau).